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Influence of the randomly varying domain length of quasi-phase-matched crystals on quadrature squeezing performance*

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During the fabrication of quasi-phase-matched (QPM) devices, errors of periodic structure are usually inevitable. The errors result in the deviation of the actual periodic domain length from the theoretical value. In this paper, we numerically analyse the influence of errors on the quadrature squeezing performance of a degenerate optical parametric amplifier consisting of QPM devices. It is shown that errors significantly degrade the squeezing degree of the quadrature squeezed light. Due to the presence of the errors, the relative phase between the signal and the pump field for obtaining the maximum squeezing depends on the propagation distance of light in the crystal and the pump power.

Keywords: quasi-phase-matched, errors of periodic structure, degenerate optical parametric amplification, quadrature squeezing

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1. Introduction

It is well known that squeezed light can be obtained through second-harmonic generation (SHG) and parametric down-conversion in $\chi^{(2)}$ media.[1,2] To achieve efficient frequency conversions, optical crystals have to be prepared to satisfy phase-matching conditions. In traditional schemes, the birefringence of a crystal is explored to accomplish the phase matching. Recently, a new type of nonlinear crystal, called a quasi-phase-matched (QPM) crystal, has attracted extensive interest.[3] In a QPM crystal, the sign of the nonlinear coefficient of each domain length in the medium is reversed periodically. The great advantage of QPM techniques is that the largest element of $\chi^{(2)}$ tensor and a much longer effectively interacting length can be utilized. Thus, a high efficiency in frequency conversion can be achieved, and correspondingly squeezed light with a high squeezing degree can be effectively generated by using QPM devices. The squeezing has been observed in QPM waveguides through parametric amplification[4,5] and SHG.[6] Our group in 2001 achieved the quadrature phase squeezing light reflected from a triply resonant optical parametric oscillator.[7] The theoretical results[8,9] show that a high degree of squeezing can be obtained with a perfectly phase-matched device.

In a practical QPM device, there are inevitable deviations between the domain length and the designed length due to the imperfect fabrication technology. Noirie *et al*[10] and Maeda *et al* [11] have considered the effect of domain errors on amplitude squeezing of SHG in a QPM device. They pointed out that the domain errors can degrade the squeezing degree. In this paper we numerically analyse the influence of the errors in domain length on the quadrature squeezing of the subharmonic light generated from a degenerate optical parametric amplifier.

2. Theoretical model

During the calculation the intensive pump field is considered as a constant classical light field. Ignoring optical losses, the equations of coupled waves in the nonlinear interaction are written as[9]

$$\frac{\mathrm{d}a(z)}{\mathrm{d}z} = g\exp[i(\Delta kz + \phi_{\mathrm{pump}} - \phi_{\mathrm{signal}} - \phi_{\mathrm{idler}})]b^{+}(z),$$

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$$\frac{\mathrm{d}b(z)}{\mathrm{d}z} = g\exp[i(\Delta kz + \phi_{\mathrm{pump}} - \phi_{\mathrm{signal}} - \phi_{\mathrm{idler}})]a^+(z), \tag{1}$$

where $a(z)$ and $b(z)$ are the operators for signal and idler fields, and ϕ_{pump}, ϕ_{signal} and ϕ_{idler} are the phases of pump, signal and idler fields, respectively. $\Delta k = k_{\mathrm{pump}} - k_{\mathrm{idler}} - k_{\mathrm{signal}}$ is the wave vector mismatch. $g = (-1)^{Int(2z/\Lambda)}\left[\frac{2\omega_{\mathrm{signal}}\omega_{\mathrm{idler}}|d^{(2)}|^2 I_{\mathrm{pump}}}{n_{\mathrm{signal}}n_{\mathrm{idler}}n_{\mathrm{pump}}\varepsilon_0 c^3}\right]^{1/2}$ is the nonlinear coupling coefficient. The superscript Int indicates taking the integer part of $2z/\Lambda$, Λ is the period length of a QPM device, $d^{(2)}$ is the effective nonlinear coefficient, c is the speed of light in vacuum, I_{pump} is the pump power, ε_0 is the dielectric constant in vacuum, and n_{signal}, n_{idler} and n_{pump} are the refractive indices for signal, idler and pump fields, respectively. For a degenerate frequency down-conversion with identical signal and idler fields, Eqs.(1) are simplified to

$$\frac{\mathrm{d}a(z)}{\mathrm{d}z} = g\exp[i(\Delta kz + \phi_{\mathrm{pump}} - 2\phi_{\mathrm{signal}})]a^+z. \tag{2}$$

According to the method proposed by Bencheikh *et al*,[9] we introduce two variables

$$\begin{aligned} p(z) =& \frac{1}{2}\left\{\exp\left[-i\left(\frac{\Delta k}{2}z + \frac{\Phi}{2}\right)\right]a(z) + \exp\left[i\left(\frac{\Delta k}{2}z + \frac{\Phi}{2}\right)\right]a^+(z)\right\},\\ q(z) =& \frac{-i}{2}\left\{\exp\left[-i\left(\frac{\Delta k}{2}z + \frac{\Phi}{2}\right)\right]a(z) - \exp\left[i\left(\frac{\Delta k}{2}z + \frac{\Phi}{2}\right)\right]a^+(z)\right\}. \end{aligned} \tag{3}$$

Combining Eqs.(2) and (3), the equations of motion of $p(z)$ and $q(z)$ are obtained:

$$\begin{aligned} \frac{\mathrm{d}p(z)}{\mathrm{d}z} =& gp(z) + \frac{\Delta k}{2}q(z),\\ \frac{\mathrm{d}q(z)}{\mathrm{d}z} =& -gq(z) - \frac{\Delta k}{2}p(z). \end{aligned} \tag{4}$$

Taking $K = [(\Delta k/2)^2 - g^2]^{1/2}$ and assuming $(\Delta k/2)^2 > g^2$ (which is usually satisfied in the practical system), we introduce the dimensionless variables

$$\varDelta = \Delta k/2K, \quad G = g/K,$$

$$C = \cos(Kz), \quad S = \sin(Kz).$$

Equations (4) are simplified to

$$\begin{bmatrix} p(z)\\ q(z) \end{bmatrix} = \begin{bmatrix} C + GS & \Delta S\\ -\Delta S & C - GS \end{bmatrix}\begin{bmatrix} p(0)\\ q(0) \end{bmatrix}. \tag{5}$$

Two typical cases are analysed as follows.

2.1. Perfect QPM device

Assuming $\Lambda = 2L$, then we have $\Lambda = 2L = \pi/K$. At the end of the first half-period ($z = L$), we have $C = \cos(\pi/2)$, $S = \sin(\pi/2)$, and thus Eq.(5) becomes

$$\begin{bmatrix} p(L)\\ q(L) \end{bmatrix} = \begin{bmatrix} G & \Delta\\ -\Delta & -G \end{bmatrix}\begin{bmatrix} p(0)\\ q(0) \end{bmatrix}. \tag{6}$$

Because the sign of g and G is reversed for the inverted domains of the QPM device, at the end of the second half-period ($z = 2L$) we have

$$\begin{bmatrix} p(2L)\\ q(2L) \end{bmatrix} = \begin{bmatrix} -G & \Delta\\ -\Delta & G \end{bmatrix}\begin{bmatrix} G & \Delta\\ -\Delta & -G \end{bmatrix}\begin{bmatrix} p(0)\\ q(0) \end{bmatrix}. \tag{7}$$

After propagating through n periods, we obtain

$$\begin{bmatrix} p(2nL)\\ q(2nL) \end{bmatrix} = \begin{bmatrix} -(G^2+\Delta^2) & -2\Delta G\\ -2\Delta G & -(G^2+\Delta^2) \end{bmatrix}^n \begin{bmatrix} p(0)\\ q(0) \end{bmatrix}. \tag{8}$$

Equation (8) can be rewritten as

$$\begin{bmatrix} p(2nL)\\ q(2nL) \end{bmatrix} = \begin{bmatrix} \frac{\lambda_1^n+\lambda_2^n}{2} & \frac{\lambda_1^n-\lambda_2^n}{2}\\ \frac{\lambda_1^n-\lambda_2^n}{2} & \frac{\lambda_1^n+\lambda_2^n}{2} \end{bmatrix}\begin{bmatrix} p(0)\\ q(0) \end{bmatrix}, \tag{9}$$

where $\lambda_1 = -(G+\Delta)^2, \lambda_2 = -(-G+\Delta)^2$.

Combining Eqs.(3) and (9), we obtain the annihilation operator of the signal field:

$$a(z) = \frac{\exp[i(\Delta kz/2 + \Phi/2)]}{2}\begin{bmatrix} a(0)\exp(-i\Phi/2)(\lambda_1^n+\lambda_2^n)+\\ ia^+(0)\exp(i\Phi/2)(\lambda_1^n-\lambda_2^n) \end{bmatrix}. \tag{10}$$

We assume the original signal wave (z=0) be in a coherent state. The variances of the quadrature amplitude and phase operators are calculated algebraically

$$\begin{aligned} X =& a + a^+,\\ <(\Delta X)^2> =& 1 + \frac{(\lambda_1^n-\lambda_2^n)^2}{2} + \frac{\lambda_1^{2n}-\lambda_2^{2n}}{2} \times\cos\left(\Delta kz + \Phi + \frac{\pi}{2}\right), \end{aligned} \tag{11}$$

$$\begin{aligned} Y =& -i(a - a^+),\\ <(\Delta Y)^2> =& 1 + \frac{(\lambda_1^n-\lambda_2^n)^2}{2} + \frac{\lambda_1^{2n}-\lambda_2^{2n}}{2} \times\cos\left(\Delta kz + \Phi - \frac{\pi}{2}\right), \end{aligned} \tag{12}$$

For the particular case of $z = 2mL = m\pi/K \approx 2m/\Delta k$, where m is an integer, we obtain $\Delta kz = 2m\pi$. Equations (11) and (12) can be written as

$$< (\Delta X)^2 >=1+\frac{(\lambda_1^n-\lambda_2^n)^2}{2}+\frac{\lambda_1^{2n}-\lambda_2^{2n}}{2} \times\cos\left(\varPhi+\frac{\pi}{2}\right). \tag{13}$$

$$< (\Delta Y)^2 >=1+\frac{(\lambda_1^n-\lambda_2^n)^2}{2}+\frac{\lambda_1^{2n}-\lambda_2^{2n}}{2} \times\cos\left(\varPhi-\frac{\pi}{2}\right). \tag{14}$$

2.2. QPM device with period error

The deviation of the domain length from the ideal length can be categorized into two types.[11] (a) Uniform deviation in the whole length of the device. In this case we have $\varLambda = \varLambda_0(1+\sigma)$, where the introduced parameter σ (deviation parameter) denotes the degree of deviation, i.e. when σ=0, there is no deviation, and a larger σ corresponds to a larger deviation. $\varLambda_0$ is the perfect QPM period length, and $\varLambda$ is the actual period length. (b) Stochastic variations of the domain length caused by non-ideal fabrication processes. Considering the stochastic variations of the period length, we write

$$\frac{1}{M}\sum_{n=1}^{M}\varLambda_n=\varLambda_0,\quad \frac{1}{M}\sum_{n=1}^{M}\left(\varLambda_n^+-\frac{\varLambda_0}{2}\right)^2=\frac{1}{4}\sigma^2\varLambda_0^2,$$

and

$$\frac{1}{M}\sum_{n=1}^{M}\left(\varLambda_n^--\frac{\varLambda_0}{2}\right)^2=\frac{1}{4}\sigma^2\varLambda_0^2,$$

where $\varLambda^+$ and $\varLambda^-$ are the lengths of the domain with positive and negative nonlinear coefficients respectively. M denotes the total period numbers included in the QPM device.

We consider a practical system with $\sigma \neq 0$, now $C = \cos\left[\frac{\pi}{2}(1+\sigma)\right]$ and $S = \sin\left[\frac{\pi}{2}(1+\sigma)\right]$ instead of the ideal case C=0 and S=1. Solving Eq.(5) numerically, we obtain

$$\begin{bmatrix} p(2nL) \\ q(2nL) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} p(0) \\ q(0) \end{bmatrix}, \tag{15}$$

where matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the product of $2n$ matrices $\begin{bmatrix} C+GS & \Delta S \\ -\Delta S & C-GS \end{bmatrix}$ which describe the propagation of the signal field.

Combining Eqs.(3) and (15), we have

$$a(z)=\frac{\exp[i(\Delta kz/2+\varPhi/2)]}{2} \times\begin{bmatrix} a(0)\exp(-i\varPhi/2)\alpha+ \\ ia^+(0)\exp(i\varPhi/2)\beta \end{bmatrix}. \tag{16}$$

where $\alpha = (A+D)+i(C-B)$, $\beta = (B+C)+i(D-A)$.

We assume that the original signal is in a coherent state, and the variances of the quadrature amplitude and phase operators are calculated algebraically

$$< (\Delta X)^2 >=\frac{1}{4}(\alpha\beta \mathrm{e}^{i(\varPhi+\pi/2)} + \alpha^*\beta^*\mathrm{e}^{-i(\varPhi+\pi/2)}+|\alpha|^2+|\beta|^2), \tag{17}$$

$$< (\Delta Y)^2 >=\frac{1}{4}(\alpha\beta \mathrm{e}^{i(\varPhi-\pi/2)} + \alpha^*\beta^*\mathrm{e}^{-i(\varPhi-\pi/2)}+|\alpha|^2+|\beta|^2). \tag{18}$$

3. Numerical analysis

As an example, we numerically calculate the variance of quadrature amplitude of the signal field generated from a degenerate parametric down-conversion in an actual periodically poled $LiNbO_3$ (PPLN). The corresponding parameters are as follows: λ_{pump}=1.06μm, λ_{signal}=λ_{idler}=2.12μm, the nonlinear coefficient $d^{(2)}$=27pm/V (d_{33}), the crystal length L_{ppln}=20mm, and the period length $\varLambda$=31μm.

3.1. The effect of uniform period deviation

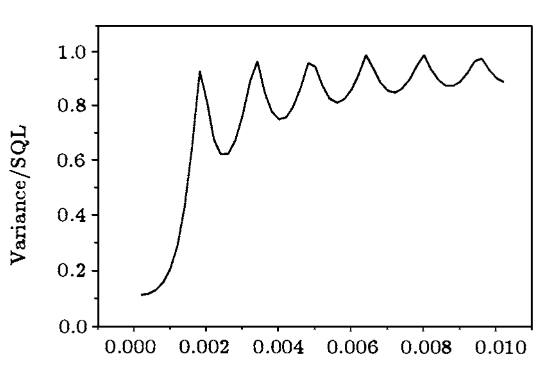

Fig.1. The variance of quadrature amplitude versus the deviation parameter σ in the case of uniform period deviation. SQL denotes the standard quantum limit (I_{pump}=1.6MW/cm^2, L_{ppln}=20mm).

The variance of quadrature amplitude as a function of the deviation parameter σ is plotted in Fig.1. As σ increases, the variance increases quickly at first, then the variance appears in periodic oscillations, and the minimum variance is much higher than that for σ=0. This means that, when $\sigma \neq 0$, the quadrature squeezing decreases greatly. It should be pointed out that uniform period deviation can be compensated by tuning the temperature of QPM devices. This is because, for a QPM crystal, we have

$$\Delta K = \Delta k - \frac{2\pi}{\Lambda}, \quad \Delta k = 2\pi\left(\frac{1}{\lambda_p} - \frac{1}{\lambda_s} - \frac{1}{\lambda_i}\right), \quad (19)$$

where ΔK is the phase mismatch, and Δk is the wave vector mismatch. In an ideal case, ΔK=0, when the period length has a uniform deviation, the phase-matching condition is destroyed: $\Delta K = \Delta k - \dfrac{2\pi}{\Lambda} \neq 0$. Since Δk is a function of temperature, so we can change Δk by tuning the temperature of the crystal and making ΔK=0 again.

3.2. The random period deviations

For random period deviations, the length of every period changes randomly. A practical QPM device usually consists of thousands of periods; typically we assume that the period lengths obey the normal distribution

$$P(\Lambda) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(\Lambda-\Lambda_0)^2}{2\sigma^2}\right]. \quad (20)$$

Figure 2 shows the variance of quadrature amplitude versus the propagation distance in a QPM device for three different deviation parameters ($\sigma = 0$, $\sigma = 0.01$, $\sigma = 0.1$). For a given σ we plot several curves: four curves for σ=0.01 and five curves for σ=0.1 (corresponding to four different crystals with deviation parameter σ=0.01 and five different crystals with deviation parameter σ=0.1). When the propagation distance increases, the variance decreases monotonically for small σ (σ=0.01) and there is no distinct difference between different crystals. However, for larger σ the curves separate distinctly and there are fluctuations in each curve. From Fig.2 we see that the larger σ has a stronger influence on the squeezing performance. Figure 3 illustrates the relative phase between the pump and the signal fields, corresponding to the minimum variance of quadrature amplitude versus the propagation distance. For perfect QPM devices (σ=0), the relative phase is a constant $\pi/2$ and does not change with the propagation distance. For σ=0.01 the relative phase presents weak fluctuations around $\pi/2$. But, for σ=0.1, the relative phase oscillates randomly between $-\pi$ and π. Figure 4 shows the variance of quadrature amplitude as a function of pump power for three different σ. The four curves for σ=0.01 and five curves for σ=0.1 correspond to the four different crystals with σ=0.01 and five different crystals with σ=0.1, respectively. As the pump power increases, the variance decreases due to a stronger nonlinear interaction, however, the rising speed reduces for larger σ. Again, the curves for σ=0.01 are very close but, for σ=0.1 the curves separate distinctly and the variance decreases slowly without oscillation. In Fig.5 we plot the relative phase between the pump and the signal fields corresponding to the minimum variance versus the pump power; for σ=0.1, it is a constant value of $\pi/2$, for σ=0.01 and σ=0.1, little change is shown.

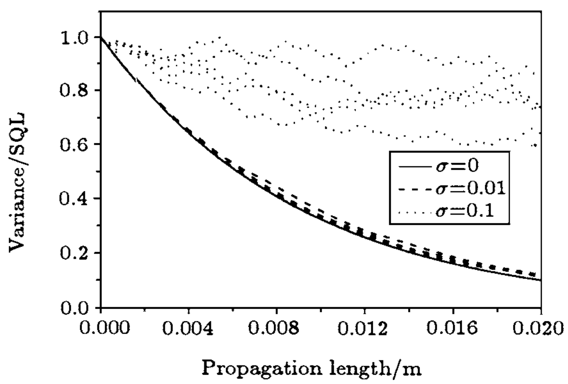

Fig.2. The variance as a function of propagation distance for different σ in the case of random period deviation (I_{pump}=1.6MW/cm^2).

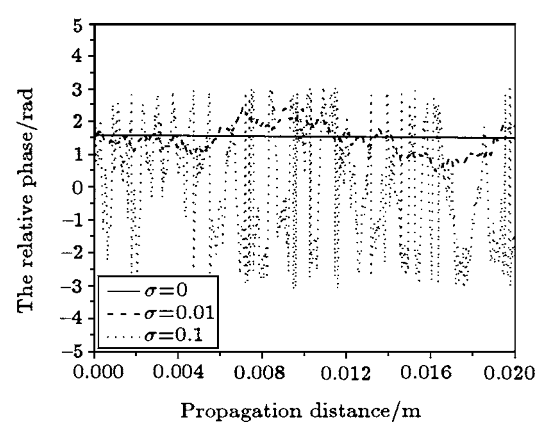

Fig.3. The relative phase between the pump and the signal fields for maximum squeezing as a function of propagation distance for given σ in the case of random period deviation (I_{pump}=1.6MW/cm^2).

The influence of σ on the quadrature amplitude squeezing is clearly seen in Fig.6. At a certain σ we plot three dots corresponding to three different crystals with the same σ. The maximum squeezing of 10dB is obtained at σ=0. The squeezing decreases as σ increases and there are fluctuations in the curve. The reduction of squeezing is 1dB for $\sigma < 0.01$, and there are no more differences between the three curves. When σ reaches 0.1, the squeezing degrades to 1dB and the three curves obviously separate. As mentioned above, for different QPM devices with the same deviation parameter σ=0.1 the squeezing performances are different.

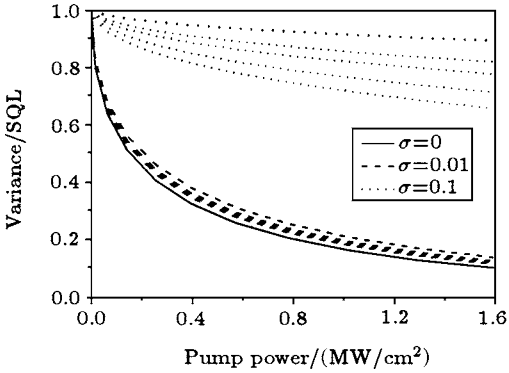

Fig.4. The variance as a function of pump power for different σ in the case of random period deviation (L_{ppln}=20mm).

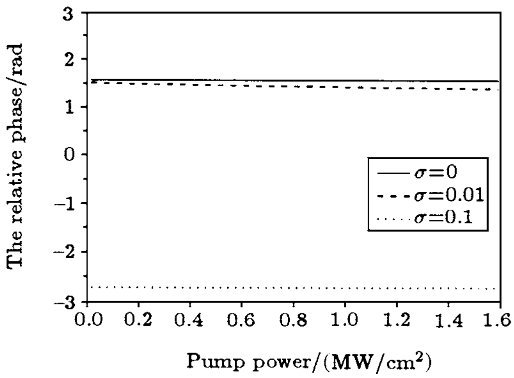

Fig.5. The relative phase between the pump and the signal fields for minimum variance versus pump power in the case of random period deviation (L_{ppln}=20mm).

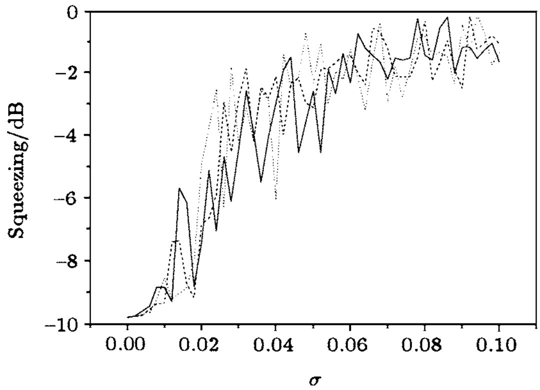

Fig.6. The squeezing as a function of the deviation parameter σ in the case of random period deviation (I_{pump}=1.6MW/cm^2, L_{ppln}=20mm).

4. Conclusion

In this paper we have studied numerically the influences of the period deviation in QPM devices on the quadrature squeezing light generated by a degenerate optical parametric amplifier. It is shown that, for uniform period errors, the quadrature squeezing degrades quickly at first, then a periodic oscillation around a higher noise level appears as the deviation σ increases. This kind of deviation can be compensated by tuning the temperature of the QPM device. For the random period errors, the dependence of the squeezing upon the deviation parameter σ and the different QPM devices with the same deviation parameter are more complicated. Generally, the larger σ has a stronger influence on the squeezing performance. We have also studied the dependence of the relative phase between the signal and the pump fields for obtaining the maximum squeezing on the deviation parameter σ, the length of QPM crystal and the pump power. We believe that the calculated results based on the actual parameters of a PPLN can provide some valuable references for the design and application of the QPM devices.

References

[1] Zhang K S, Zhang J, Xie C D and Peng K C 2000 *Acta Phys. Sin.* **49** 80 (in Chinese)
[2] Peng K C, Pan Q, Wang H, Zhang Y, Su H and Xie C D 1998 *Appl. Phys.* B **66** 755
[3] Zhu S N, Zhu Y Y and Ming N B 1997 *Science* **278** 843
[4] Anderson E, Beck M, Raymer M G and Bieclein J D 1995 *Opt. Lett.* **20** 620
[5] Serkland D K, Fejer M M, Byer R L and Yamamoto Y 1995 *Opt. Lett.* **20** 1649
[6] Serkland D K, Kumar P, Arbore M A and Fejer M M 1997 *Opt. Lett.* **22** 1497
[7] Li Y M, Fan Q Y, Zhang K S, Xie C D and Peng K C 2001 *Acta Phys. Sin.* **52** 1492 (in Chinese)
[8] Chickarmane V S and Agarwal G S 1998 *Opt. Lett.* **23** 1132
[9] Bencheikh K, Huntziger E and Levenson J A 1995 *J. Opt. Soc. Am.* B **12** 847
[10] Noirie L, Bidakovic P and Levenson J A 1997 *J. Opt. Soc. Am.* B **14** 1
[11] Maeda J, Matsuda I and Fukuchi Y 2000 *J. Opt. Soc. Am.* B **17** 942